Dust ion-acoustic K-dV and modified K-dV solitons in a dusty degenerate dense plasma

K. N. Mukta, M. S. Zobaer, N. Roy, A. A. Mamun
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

A theoretical investigation has been made of the roles of the degeneracy and the dynamics of electrons and ions on the DIA (dust ion-acoustic) Korteweg-de Vries and modified Korteweg-de Vries solitons that are found to exit in a dusty degenerate dense plasma containing non-relativistic degenerate ions and both non-relativistic and ultra relativistic electrons fluids, and the negatively charged dust grains. This fluid model, which is valid for both the non-relativistic and ultra-relativistic limits has been employed with the reductive perturbation method. The K-dV and modified K-dV equations have been derived, and numerically examined. The basic features of K-dV and modified K-dV solitons have been analyzed. It has been observed that the dusty degenerate plasma system under consideration supports the propagation of silitons obtained from the solutions of K-dV and modified K-dV equations. The relevance of our results obtained from this investigation in compact astrophysical objects is briefly discussed.

I. INTRODUCTION

Recently, the physics of dusty plasma is receiving a great deal of attention[1–3]. Dusty plasmas are characterized as a low temperature multispecies ionized gas comprising electrons, protons, and negatively (or positively) charged grains of micrometer or submicrometer size. The study of the collective effects in dusty plasmas is of significant interest. Charged dust grains are found to modify or even dominate wave propagation [4–8], wave scattering [9–12], wave instability [13], self-similar plasma expansion [14], velocity modulation [15], charged particle transport [16], and ion trapping [17]. However, most of the studies on wave motions [4–15]in dusty plasma assume constant charge on the dust grains.

Now-a-days a number of authors have become interested to study the properties of matter under extreme conditions [18–23], which occur due to the combine effect of Pauli's exclusion principle and Heisenberg's uncertainty principle, depends only on the number density of constituent particles, but independent on it's own temperature [34–36]. This degenerate pressure has an important role to study the electrostatic perturbation in matters which exist in extreme conditions [18–20, 37, 38]. Electron degenerate pressure will halt the gravitational collapse of a star if its mass is below the Chandrasekhar limit (i.e. 1.44 solar masses) [39]. This is the pressure that prevents a white dwarf star from collapsing. Astrophysical aspects of high density like in many cosmic environments, compact astrophysical objects [24–27] and planetary systems have been recently discussed by Forton [33]. Examples of the latter are white and brown dwarf stars [28–30], as well as massive Jupiter [31] which serves as the super-Earth terrestrial planets around other stars [32], and the benchmark for giant planets.

In case of such a compact object the degenerate electron number density is so high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, even more [34–36]) that the electron Fermi energy is comparable to the electron mass energy and as a result the electron speed becomes comparable to the speed of light in vacuum. For such interstellar compact objects the equation of state for degenerate ions and electrons are mathematically explained by Chandrasekhar [20] for two limits, named as non-relativistic and ultra-relativistic limits. Chandrasekhar [18, 20] presented a general expression for the relativistic ion and electron pressures in his classical papers. The pressure for ion fluid can be given by the following equation

$$P_i = K_i n_i^{\alpha}, \tag{1}$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{3}$$

where

$$\gamma = \alpha; K_e = K_i \text{ for nonrelativistic limit, and} \tag{4}$$

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{5}$$

in the ultra-relativistic limit [18–20, 34, 35].

Recently, a large number of authors [34, 35, 40–50], etc. have used the pressure laws (3) to (5) investigate the linear and nonlinear properties of electrostatic and electromagnetic waves, by using the non-relativistic quantum hydrodynamic (QHD) [40] and quantum-magnetohydrodynamic(Q-MHD) [43] models and by assuming either immobile ions or non-degenerate uncorrelated mobile ions. It turns out that the presence of the latter and degenerate ultra relativistic electrons with the pressure law (3-5) admits one-dimensional localized ion models (IMs) supported by linear and non linear ion inertial forces and the pressure of degenerate electron fluids in a dense quantum plasma that is unmagnetized. Furthermore, modified Volkov solutions of the Dirac equation for electrostatic and electromagnetic waves in relativistic quantum plasmas have been discussed by Mendonca and Serbeto [51]. Again in this present days, some authors [52–54] has made a number of theoretical investigations on the nonlinear propagation of electrostatic waves in degenerate quantum plasma. These investigations are mainly based on the electron equation of state, which are only valid for the non-relativistic limit. Some investigations have been also made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma which are mainly based on the degenerate electron equation of state valid for ultra-relativistic limit [34–36].

Still now, there is no theoretical investigation has been made to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits on the propagation of electrostatic solitary waves in a dusty degenerate dense plasma system. Therefore, in our paper we study the properties of the solitons considering a dusty degenerate dense plasma containing degenerate electron-ion fluid (both non-relativistic and ultra-relativistic limits) with the arbitrary (either positive or negative) charged dust grains to study the basic features of the electrostatic solitary structures with the solutions of modified K-dV equation. Our considered model is relevant to compact interstellar objects (i.e. white dwarf, neutron star, black hole, etc.).

II. GOVERNING EQUATIONS

We consider a one-dimensional, unmagnetized dusty degenerate electro-ion plasma system containing non-relativistic degenerate cold ion and both non-relativistic and ultra-relativistic degenerate electron fluids with arbitrary charged dust grains. We are interested in the propagation of electrostatic perturbation in such a dusty degenerate dense plasma. Thus, at equilibrium condition we have $n_{i0} = n_{e0}$, where n_{i0} (n_{e0}) is the ion (electron) number density at equilibrium. The nonlinear dynamics of the electrostatic waves propagating in such a degenerate plasma is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{6}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^\alpha}{\partial x} = 0, \tag{7}$$

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^\gamma}{\partial x} = 0, \tag{8}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\rho, \tag{9}$$

$$\rho = n_i - n_e(1-\mu). \tag{10}$$

where n_i (n_e) is the ion (electron) number density normalized by its equilibrium value n_{i0} (n_{e0}), u_i is the ion fluid speed normalized by $C_i = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass, c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$, the space variable ($x$) is normalized by $\lambda_s = (m_e c^2/4\pi n_0 e^2)^{1/2}$ and p is the polarity of the dust grains. The constants are $K_1 = n_0^{\alpha-1} K_i/{m_i}^2 {C_i}^2$ and $K_2 = n_0^{\gamma-1} K_e/m_i {C_i}^2$.

III. DERIVATION OF K-DV EQUATION

Now we derive a dynamical K-dV equation for the nonlinear propagation of the DIA waves by using equations (6-10). To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic dusty degenerate dense plasma due to the effect of dispersion, we first introduce the stretched coordinates [55]

$$\zeta = \epsilon^{1/2}(x - V_p t), \tag{11}$$

$$\tau = \epsilon^{3/2} t. \tag{12}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_i, n_e, u_i, ρ, and ϕ, in power series of ϵ:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{13}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{14}$$

$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{15}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{16}$$

$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \tag{17}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (6-10), using equations (11-17), give as $u_i^{(1)} = V_p \phi^{(1)}/(V_p^2 - K_1')$, $n_i^{(1)} = \phi^{(1)}/(V_p^2 - K_1')$, $n_e^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{K_2'/(1-\mu) + K_1'}$ where $K_1' = \alpha K_1/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$. The relation $V_p = \sqrt{K_2'/(1-\mu) + K_1'}$ represents the dispersion relation for the dust ion-acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dispersive dust ion-acoustic type electrostatic waves in a degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \xi}[u_i^{(2)} + n_i^{(1)} u_i^{(1)}] = 0, \tag{18}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \xi} - u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi}$$
$$-K_1' \frac{\partial}{\partial \xi}[n_i^{(2)} + \frac{(\alpha-2)}{2}(n_i^{(1)})^2] = 0, \tag{19}$$

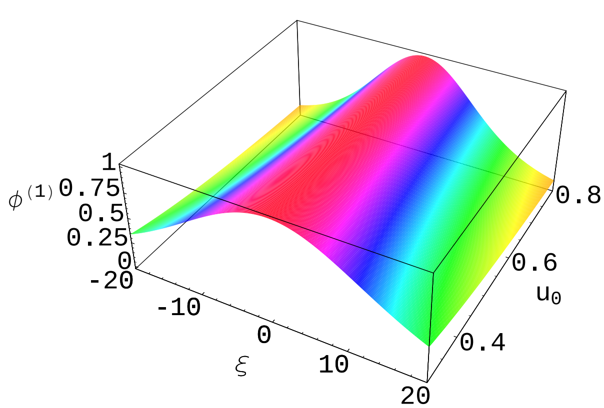

FIG. 1: Showing the effect of u_0 on soliton (potential structure) obtained from eq. (26) for both electron-ion being non-relativistic degenerate when μ is 0.5.

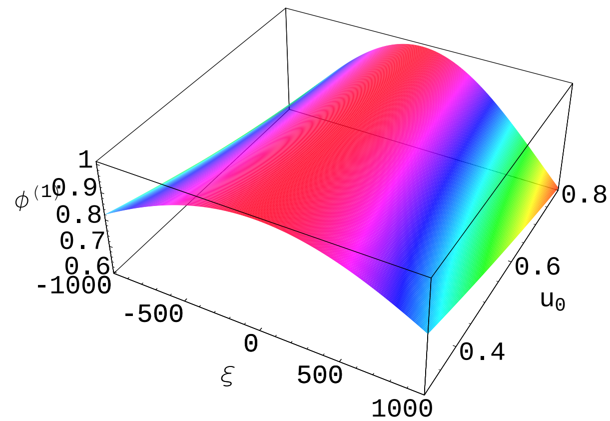

FIG. 2: Showing the effect of u_0 on soliton (potential structure) obtained from eq. (26) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when μ is 0.5.

$$\frac{\partial \phi^{(2)}}{\partial \xi} - K_2' \frac{\partial}{\partial \xi}\left[n_e^{(2)} + \frac{(\gamma-2)}{2}\left(n_e^{(1)}\right)^2\right] = 0, \quad (20)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = -\rho^{(1)}, \quad (21)$$

$$\rho^{(1)} = n_i^{(2)} - (1-\mu) n_e^{(2)}. \quad (22)$$

Now, combining equations (18-22) we deduce a Korteweg-de Vries equation as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \quad (23)$$

where

$$A = \frac{\left(V_p^2 - K_1'\right)^2}{2V_p}\Big[\frac{3V_p^2 + K_1'(\alpha-2)}{\left(V_p^2 - K_1'\right)^3} + \frac{(1-\mu)(\gamma-2)}{{K_2'}^2}\Big], \quad (24)$$

$$B = \frac{\left(V_p^2 - K_1'\right)^2}{2V_p}. \quad (25)$$

The stationary solitary wave solution of equation (23) is

$$\phi^{(1)} = \phi_m \mathrm{sech}^2\left(\frac{\xi}{\Delta}\right), \quad (26)$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude, $\phi_m = 3u_0/A$, and the width, $\Delta = (4B/u_0)^{1/2}$.

IV. DERIVATION OF MODIFIED K-DV EQUATION

To examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect

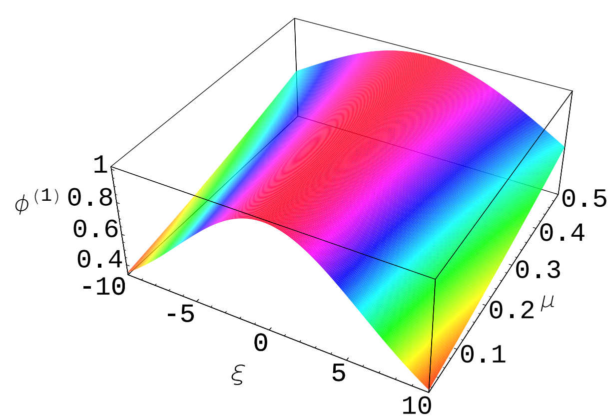

FIG. 3: Showing the effect of μ on soliton (potential structure) obtained from eq. (26) for both electron-ion being non-relativistic degenerate when u_0 is 0.1.

of dispersion by analyzing the outgoing solutions of equations (6-10), we now introduce the new set of stretched coordinates for the modified K-dV equation is:

$$\xi = \epsilon(x - V_p t), \quad (27)$$

$$\tau = \epsilon^3 t. \quad (28)$$

To the lowest order in ϵ, using equations (27,28, and 13-17), into the equations (6-10), we find the same results as we have had for the solitons for K-dV equation.

To the next higher order in ϵ, we obtain a set of equations, which, after using the values of $u_i^{(1)}$, $n_i^{(1)}$, and $n_e^{(1)}$, can be simplified as

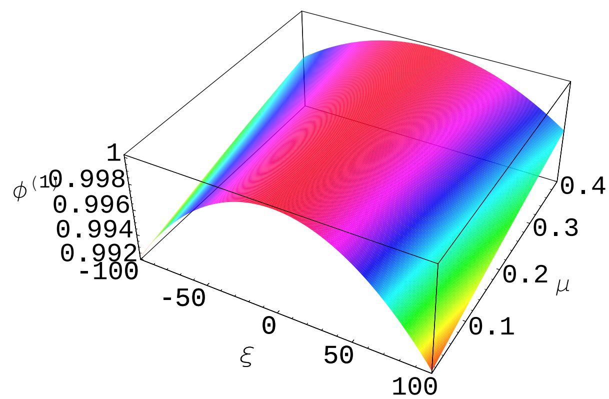

FIG. 4: Showing the effect of μ on soliton (potential structure) obtained from eq. (26) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when u_0 is 0.1.

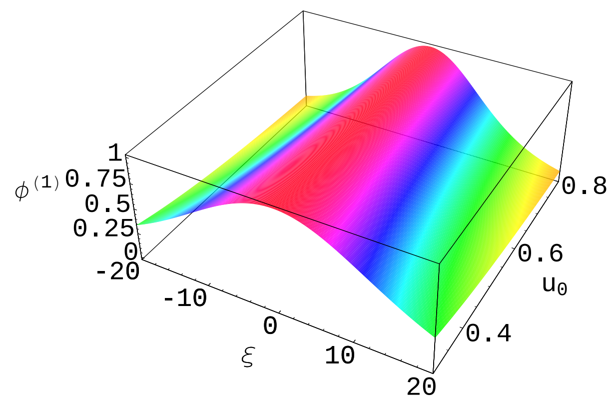

FIG. 5: Showing the effect of u_0 on soliton (potential structure) obtained from eq. (41) for both electron-ion being non-relativistic degenerate when μ is 0.5.

$$u_i^{(2)} = \frac{V_p\phi^{(2)}}{V_p^2K_1'} + [V_pK_1' + \frac{V_p^3}{2} + \frac{K_1V_p(\alpha-2)}{2}] \frac{(\phi^{(1)})^2}{(V_p^2-K_1')^3}, \quad (29)$$

$$n_i^{(2)} = \frac{\phi^{(2)}}{V_p^2-K_1'} + [3V_p^2 + K_1'(\alpha-2)] \frac{(\phi^{(1)})^2}{2(V_p^2-K_1')^3}, \quad (30)$$

$$n_e^{(2)} = \frac{\phi^{(2)}}{K_2'} - \frac{(\gamma-2)(\phi^{(1)})^2}{2(K_2')^2}, \quad (31)$$

$$\rho^{(2)} = \frac{1}{2}\mathring{A}\phi^{(2)} \quad (32)$$

where

$$\mathring{A} = \frac{(\gamma-2)(1-\mu)}{(K_2')^2} + \frac{3V_p^2 + K_2'(\alpha-2)}{(V_p^2-K_1')^3}, \quad (33)$$

To further higher order of ϵ, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial\tau} - V_p\frac{\partial n_i^{(3)}}{\partial\xi} + \frac{\partial u_i^{(3)}}{\partial\xi} + \frac{\partial}{\partial\xi}[u_i^{(2)}n_i^{(1)} + n_i^{(2)}u_i^{(1)}] = 0, \quad (34)$$

$$\frac{\partial u_i^{(1)}}{\partial\tau} - V_p\frac{\partial u_i^{(3)}}{\partial\xi} + \frac{\partial}{\partial\xi}[u_i^{(1)}u_i^{(2)}] + \frac{\partial\phi^{(3)}}{\partial\xi} + K_1'\frac{\partial}{\partial\xi}[n_i^{(3)} + (\alpha-2)(n_i^{(1)}n_i^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_i^{(1)})^3] = 0, \quad (35)$$

$$\frac{\partial\phi^{(3)}}{\partial\xi} - K_2'\frac{\partial}{\partial\xi}[n_e^{(3)} + (\alpha-2)(n_e^{(1)}n_e^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_e^{(1)})^3] = 0, \quad (36)$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = -\rho^{(3)}, \quad (37)$$

$$\rho^{(3)} = n_i^{(3)} - (1-\mu)n_e^{(3)}. \quad (38)$$

Now combining equations (34) - (38) and using the values of $n_i^{(1)}$, $n_i^{(2)}$, $u_i^{(1)}$, $u_i^{(2)}$, and $\rho^{(2)}$, we obtain an equation of the form

$$\frac{\partial\phi^{(1)}}{\partial\tau} + \beta\{\phi^{(1)}\}^2\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \quad (39)$$

where the value of B is as before and β is given by

$$\beta = \frac{1}{2(V_p^2-K_1')^5}[K_1'(\alpha-2)(\alpha-3)(V_p^2-K_1') + 15V_p^4 + 12K_1'V_p^2 + 18K_1'V_p^2(\alpha-2)^2 + 3(K_1'(\alpha-2))^2] - \frac{(1-\mu)(12-10\gamma+2\gamma^2)}{2(K_2')^3}, \quad (40)$$

We call equation (39) as modified K-dV equation for planner geometry. The stationary solitary solution of equation (39) is given by

$$\phi^{(1)} = \phi_m sech(\frac{\xi}{\Delta}), \quad (41)$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude is $\phi_m = \sqrt{\frac{6u_0}{\beta}}$, the width is $\Delta = \sqrt{\frac{1}{\gamma\phi_m}}$ and u_0 is the plasma species speed at equilibrium.

V. NUMERICAL ANALYSIS

In the figures 1-4 we have tried to show the solitons (solitary profiles) from the solution of K-dV equation (26)

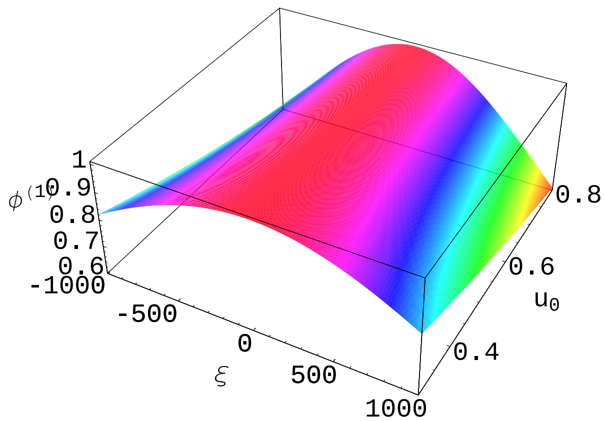

FIG. 6: Showing the effect of u_0 on soliton (potential structure) obtained from eq. (41) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when μ is 0.5.

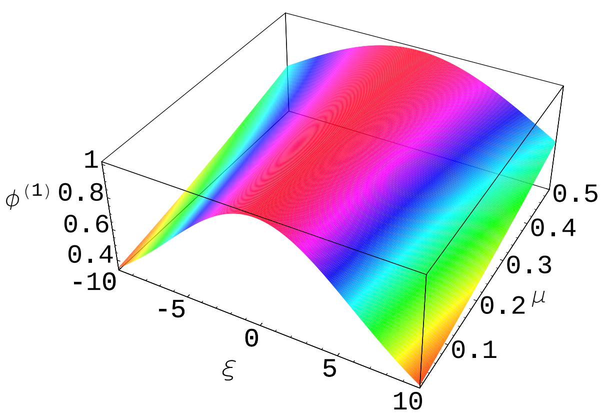

FIG. 7: Showing the effect of μ on soliton (potential structure) obtained from eq. (41) for both electron-ion being non-relativistic degenerate when u_0 is 0.1.

due to the effect of μ and u_0 on the potential, $\phi^{(1)}$ for the case of electron-ion being non-relativistic degenerate and ion being non-relativistic degenerate and electron being ultra-relativistic degenerate. And the figures 5-8 represent the solitons (solitary profiles) from the solution of modified K-dV equation (41) due to the effect of μ and u_0 on the potential, $\phi^{(1)}$ for the both case of relativistic limit. From the figures 1-2 we have observed the effect of u_0 on the potential, $\phi^{(1)}$ for the solitary profiles obtained from the solution of K-dV equation (26) when we have considered the value of μ as 0.5 for the both case of relativistic limits. And from the figures 3-4 we have observed the effect of μ on the potential, $\phi^{(1)}$ for the solitary profiles obtained from the solution of K-dV equation

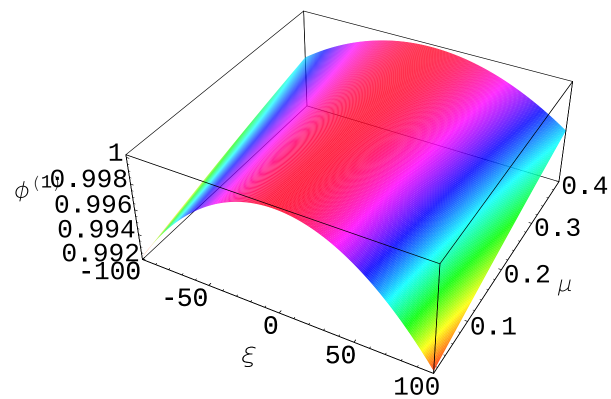

FIG. 8: Showing the effect of μ on soliton (potential structure) obtained from eq. (41) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when u_0 is 0.1.

(26) when u_0 is 0.1 for the both case of relativistic limits. Again from the figures 5-6 we have analyzed the effect of u_0 when μ is 0.5 and from the figures 7-8 we have observed the effect of μ when u_0 is 0.1 on the potential, $\phi^{(1)}$ of the solitons obtained from the solution of modified K-dV equation (41) for both case of relativistic limits.

By the careful observation on the figures 1-8 it has become clear that the terms u_0 and μ have an great effect on the potential, $\phi^{(1)}$ of the K-dV and modified K-dV solitons. Because the potential, $\phi^{(1)}$ increases more rapidly for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate than for both electron-ion being non-relativistic degenerate. Again in the same case (either ion being non-relativistic degenerate and electron being ultra-relativistic degenerate or electron-ion both being non-relativistic degenerate) the width, Δ of the solitons obtained from the solutions of K-dV and modified K-dV equations (26) and (41) decreases sharply in all conditions whatever u_0 or μ increases with the term ξ.

VI. DISCUSSION

To summarize, we have carried out solitons by deriving the K-dV and modified K-dV equations for a planar geometry in an unmagnetized plasma system containing degenerate electrons (non-relativistic or ultra relativistic limits) and degenerate ions being non-relativistic limit and the arbitrary charged dust grains. We have shown the existence of compressive (hump shape) and rarefactive (dip shape) DIA modified K-dV solitons. We have identified the basic features of potential DIA solitons, which are found to exist beyond the K-dV limit. Generally the DIA modified K-dV solitons are completely different from the K-dV solitary waves. The plasma system under consideration supports finite potential modified K-dV solitons, whose basic features depend much on the degenerate pressure of ion and electron and the presence of arbitrary charged dust grains. It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of electrostatic disturbances in laboratory plasma conditions. Our investigation would also be useful to study the effects of degenerate pressure in interstellar and space plasmas [56], particularly in stellar polytropes [57], hadronic matter and quark-gluon plasma [58], protoneutron stars [59], dark-matter halos [60] etc. Further it can be said that the analysis of shock structures, vortices, double-layers etc. in a nonplanar geometry where the degenerate pressure can play the significant role, are also the problems of great importance but beyond the scope of the present work. To conclude, we propose to perform a laboratory experiment which can study such special new features of the DIA solitons propagating in dusty plasma in presence of degenerate electrons and ions.

[1] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[2] U. de Angelis, Phys. Scr. **45**, 465 (1992).
[3] T. G. Northrop, Phys. Scr. **45**, 475 (1992).
[4] U. de Angelis, V. Formisano and M. Giordano, J. Plasma Phys. **40**, 399 (1988).
[5] N. N. Rao, P. K. Shukla and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[6] N. D'Angelo, Planet. Space Sci. **38**, 1143 (1990).
[7] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[8] F. Verheest, Planet. Space Sci.**40**, 1 (1992).
[9] N. D'Angelo and B. Song , Planet. Space Sci. **38**, 1577 (1990).
[10] U. de Angelis, A. Forlani, V. N. Tsytovich and R. Bingham, J. Geophys. Res. **97**, 6261 (1992).
[11] C. La Hoz, Phys. Scr. **45**, 529 (1992).
[12] T. Hagfors, J. Atoms. Terr. Phys. **54**, 333 (1992).
[13] P. K. Shukla and L. Stenflo, Astrophys. Space Sci. **190**, 23 (1992).
[14] K. E. Lonngren, Space Sci. **38**, 1457 (1990).
[15] K. E. Lonngren, Space Sci. **40**, 763 (1992).
[16] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neil and D. K. Coultas, Phys. Lett. **68**,313 (1992).
[17] J. Goree, Phys. Rev. Lett. **69**, 277 (1992).
[18] S. Chandrasekhar, Phi. Mag. **11**, 592 (1931).
[19] S. Chandrasekhar, Astrophys. J. **74**, 81 (1931).
[20] S. Chandrasekhar, Mon. Not. R. Astron. Soc. **170**, 405 (1935).
[21] S. Chandrasekhar, *An Introduction to the Study of stellar structure* (Dover Publications, 1939).
[22] S. Chandrasekhar, Phys. Rev. Lett. **12**, 144 (1964).
[23] S. Chandrasekhar and R. F. Tooper, Astrophys. J. **139**, 1396 (1964).
[24] F. C. Michel, Rev. Mod. Phys. **54**, 1 (1982).
[25] H. R. Miller and P. J. Witta, *Active Galactic Nuclei* (Springer, New York, 1987),
[26] E. Tandberg-Hansen and A. G. Emslie, *The Physics of Solar Flares* (Cambridge University Press, Cambridge, 1988),
[27] M. J. Rees, *The Very Early Universe* (Cambridge University Press, Cambridge1983).
[28] M. Harwit, *Astrophysical Concepts* (John Wiley and Sons, New York, 1973), Chapter 8.
[29] H. Gursky, *in Forntiers of AstroPhyiscs* (Harvard University Press, London, 1976).
[30] S. L. Shapiro and S. A. Teukolsky, *Black holes, White dwarfs and neutron Stars: The Physics of Compact Objects* (John Wiley and Sons, New York, 1983).
[31] D. Saumon and T. Guillot, Astrophys. J. **609**, 1170 (2004).
[32] J. J. Fortney, S. H. Glenzer, M. Koening, *et al.*, Phys. Plasma **16**, 041103 (2009).
[33] V. E. Fortov, Phys. Usp. **52**, 615 (2009).
[34] A. A. Mamun and P. K. Shukla, Phys. Lett. A **324**, 4238 (2010).
[35] A. A. Mamun and P. K. Shukla, Phys. Plasmas **17**, 104504 (2010).
[36] N. Roy, S. Tasnim and A. A. Mamun, Phys. Plasmas **19**, 033705 (2012).
[37] D. Koester and G. Chanmugam, Rep. Prog. Phys. **53** (1990).
[38] E. Garcia-Berro, S. Torres, L. G. Althaus, I. Renedo, P. Lorén-Aguiltar, A. H. Córsico R. D. Rohrmann, M. Salaris and J. Isern, Nature **465**, 194 (2010).
[39] P. A. Mazzali, F. K. Rpke, S. Benetti and W. Hillebrandt, Science **315**, 5813 (2007).
[40] G. Manfredi, Fields Inst. Commun. **46**, 263 (2005).
[41] P. K. Shukla, Phys. Lett. A **352**, 242 (2006).
[42] P. K. Shukla and L. Stenflo, Phys. Lett. A **355**, 378 (2006).
[43] G. Brodin and M. Marklund, New J. Phys. **9**, 227 (2007).
[44] G. Brodin and M. Marklund, Phys. Plasmas **14**, 412607 (2007).
[45] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).
[46] M. Marklund, B. Eliasson and P. K. Shukla, Phys. Rev. E **76**, 067401 (2007).
[47] P. K. Shukla, Nature Phys. **5**, 92 (2009).
[48] P. K. Shukla and B. Eliasson, Phys. Usp. **53**, 51 (2010).
[49] W. Masood, B. Eliasson and P. K. Shukla, Phys. Rev. E **81**, 066401 (2010).
[50] W. Masood and B. Eliasson, Phys. Plasmas **18**, 034503 (2011).
[51] J. T. Mendonca and A. Serbeto, Phys. Rev. E **83**, 026406 (2011).
[52] F. Hass, Phys. Plasmas **13**, 042309 (2007).
[53] A. Misra and S. Samanta, Phys. Plasmas **15**, 123307 (2008).
[54] A. P. Misra, S. Banerjee, F. Haas, P. K. Shukla and L. P. G. Assis, Phys. Plasmas **17**, 032307 (2010).
[55] S. Maxon and J. Viecelli, Phys. Rev. Lett. **32**, 4 (1974).
[56] F. Ferro, A. Lavagno and P. Quarati, Eur. Phys. J. A **21**, 529 (2004).
[57] A. R. Plastino and A. Plastino, Phys. Lett. A **174**, 384 (1993).

[58] G. Gervino, A. Lavagno and D. Pigato, Central Euro. J. Phys., in press, DOI: 10.2478/s11534-011-0123-3, (2012).
[59] A. Lavagno and D. Pigato, Euro. Phys. J. A **47**, 52 (2011).
[60] C. Feron and J. Hjorth, Phys. Rev. E **77**, 022106 (2008).